University of Mumbai

No. AAMS(UG)/ 97 of 2021-22

CIRCULAR:-

Attention of the Principals of the Affiliated Colleges and Directors of the Recognized Institutions in Faculties of Humanities and Science & Technology.

They are hereby informed that the recommendations made by the Board of Studies in Mathematics at its online meeting held on 23rd April, 2021 vide Item No. 1 (i) and subsequently passed by the Board of Deans at its online meeting held on 11th June, 2021 vide item No. 6.17 (R) have been accepted by the Academic Council at its meeting held on 29th June, 2021 vide item No. 6.17(R) and that in accordance therewith, Finalize the proposed syllabus of S.Y.B.Sc./S.Y.B.A. (Sem-III & IV) in Mathematics under (CBCS) in 75:25 pattern has been brought into force with effect from the academic year 2021-22 accordingly. (The same is available on the University's website www.mu.ac.in).

MUMBAI – 400 032
8th October, 2021

(Dr. B.N.Gaikwad)
I/c REGISTRAR

To

The Principals of the Affiliated Colleges and Directors of the Recognized Institutions in Faculties of Humanities and Science & Technology.

A.C/6.17 (R) 29/06/2021

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No. AAMS(UG)/ 97 -A of 2021-22 MUMBAI-400 032 8th October, 2021

Copy forwarded with Compliments for information to:-

1) The Dean, Faculties of Humanities and Science & Technology
2) The Chairman, Board of Studies in Mathematics,
3) The Director, Board of Examinations and Evaluation,
4) The Director, Board of Students Development,
5) The Co-ordinator, University Computerization Centre,

(Dr. B.N.Gaikwad)
I/c REGISTRAR

Copy to :-

1. The Deputy Registrar, Academic Authorities Meetings and Services (AAMS),
2. The Deputy Registrar, College Affiliations & Development Department (CAD),
3. The Deputy Registrar, (Admissions, Enrolment, Eligibility and Migration Department (AEM),
4. The Deputy Registrar, Research Administration & Promotion Cell (RAPC),
5. The Deputy Registrar, Executive Authorities Section (EA),
6. The Deputy Registrar, PRO, Fort, (Publication Section),
7. The Deputy Registrar, (Special Cell),
8. The Deputy Registrar, Fort/ Vidyanagari Administration Department (FAD) (VAD), Record Section,
9. The Director, Institute of Distance and Open Learning (IDOL Admin), Vidyanagari,

They are requested to treat this as action taken report on the concerned resolution adopted by the Academic Council referred to in the above circular and that on separate Action Taken Report will be sent in this connection.

1. P.A to Hon'ble Vice-Chancellor,
2. P.A Pro-Vice-Chancellor,
3. P.A to Registrar,
4. All Deans of all Faculties,
5. P.A to Finance & Account Officers, (F.& A.O),
6. P.A to Director, Board of Examinations and Evaluation,
7. P.A to Director, Innovation, Incubation and Linkages,
8. P.A to Director, Board of Lifelong Learning and Extension (BLLE),
9. The Director, Dept. of Information and Communication Technology (DICT) (CCF & UCC), Vidyanagari,
10. The Director of Board of Student Development,
11. The Director, Department of Students Walfare (DSD),
12. All Deputy Registrar, Examination House,
13. The Deputy Registrars, Finance & Accounts Section,
14. The Assistant Registrar, Administrative sub-Campus Thane,
15. The Assistant Registrar, School of Engg. & Applied Sciences, Kalyan,
16. The Assistant Registrar, Ratnagiri sub-centre, Ratnagiri,
17. The Assistant Registrar, Constituent Colleges Unit,
18. BUCTU,
19. The Receptionist,
20. The Telephone Operator,
21. The Secretary MUASA

for information.

AC - _____

Item No. _____

UNIVERSITY OF MUMBAI

Syllabus

for the

Program : S.Y.B.Sc./ S.Y.B.A Sem. III & IV (CBCS)

Course : Mathematics

(Choice Based and Credit System with effect from the academic year 2021-22)

AC _____

Item No._____

UNIVERSITY OF MUMBAI

Syllabus for Approval

Sr. No.	Heading	Particulars
1	Title of the Course	S. Y. B. Sc. /B. A. Mathematics, Sem III & IV
2	Eligibility for Admission	As per university regulations
3	Passing Marks	40% (Internal 10/25 Marks and External 30/75)
4	Ordinances / Regulations (if any)	-
5	No. of Years / Semesters	Three Years / Six Semesters Programme (Syllabus for sem III & IV)
6	Level	UG
7	Pattern	Semester
8	Status	Revised
9	To be implemented from Academic Year	From Academic Year : 2021-2022

Date: 19.05.2021

Name: Prof. R. P. Deore

Signature: 19.05.2021

Chairman of BoS of Mathematics

Dr. Anuradha Majumdar (Dean, Science and Technology)
Prof. Shivram Garje (Associate Dean, Science)
Prof. R. P. Deore , Chairman (BoS) Member(BoS)
Prof. P. Veeramani, Member
Prof. S. R. Ghorpade , Member
Prof. Ajit Diwan, Member
Dr. Sushil Kulkarni, Member
Dr. S. A. Shende, Member
Prof. V. S. Kulkarni
Dr. Sanjeevani Gharge, Member
Dr. Mittu Bhattacharya, Member
Dr. Abhaya Chitre, Member
Dr. S. Aggarwal, Member
Dr. Amul Desai, Member

CONTENTS

1. Preamble

The University of Mumbai has brought into force the revised syllabi as per the Choice Based Credit System (CBCS) for the Second year B. Sc / B. A. Programme in Mathematics from the academic year 2021-2022. Mathematics has been fundamental to the development of science and technology. In recent decades, the extent of application of Mathematics to real world problems has increased by leaps and bounds. Taking into consideration the rapid changes in science and technology and new approaches in different areas of mathematics and related subjects like Physics, Statistics and Computer Sciences, the board of studies in Mathematics with concern of teachers of Mathematics from different colleges affiliated to University of Mumbai has prepared the syllabus of S.Y.B. Sc. / S. Y. B. A. Mathematics. The present syllabi of S. Y. B. Sc. for Semester III and Semester IV has been designed as per U. G. C. Model curriculum so that the students learn Mathematics needed for these branches, learn basic concepts of Mathematics and are exposed to rigorous methods gently and slowly. The syllabi of S. Y. B. Sc. / S. Y. B. A. would consist of two semesters and each semester would comprise of three courses and one practical course for S. Y. B. Sc Mathematics and two courses and one practical course for each semester for S. Y. B. A. Mathematics.

Aims and Objectives :

(1) Give the students a sufficient knowledge of fundamental principles, methods and a clear perception of innumerous power of mathematical ideas and tools and know how to use them by modeling, solving and interpreting.

(2) Reflecting the broad nature of the subject and developing mathematical tools for continuing further study in various fields of science.

(3) Enhancing students' overall development and to equip them with mathematical modeling abilities, problem solving skills, creative talent and power of communication necessary for various kinds of employment.

(4) A student should get adequate exposure to global and local concerns that explore them many aspects of Mathematical Sciences

2. Programme Outcomes:

(1) Enabling students to develop positive attitude towards mathematics as an interesting and valuable subject Enhancing students overall development and to equip them with
(2) mathematical modeling, abilities, problem solving skills, creative talent and power of communication. Acquire good knowledge and understanding in advanced areas of
(3) mathematics and statistics.

3. Course outcomes:

1. Calculus (Sem III) & Multivariable Calculus I(Sem IV): This course gives introduction to basic concepts of Analysis with rigor and prepares students to study further courses in

Analysis. Formal proofs are given lot of emphasis in this course which also enhances understanding of the subject of Mathematics as a whole.

2. Linear Algebra I (Sem III) & Linear Algebra II (Sem IV): This course gives expositions to system of linear equations and matrices, Vector spaces, Basis and dimension, Linear Transformation, Inner product space, Eigen values and eigenvectors.

3. Ordinary Differential Equations (Sem III) prepares learner to get solutions of so many kinds of problems in all subjects of Science and also prepares learner for further studies of differential equations and related fields.

4. Numerical Methods and Statistical Methods: Lerner will learn different types of Numerical methods and statistical methods to apply in different fields of Mathematics.

(UNIVERSITY OF MUMBAI)

Syllabus for: S.Y.B.Sc./S.Y.B.A.
Program: B.Sc./B/A.
Course: Mathematics
Choice based Credit System (CBCS)
with effect from the
academic year 2021-22

4. Course structure with minimum Credits and Lectures/ Week

SEMESTER III

Calculus III				
Course Code	UNIT	TOPICS	Credits	L/Week
USMT 301, UAMT 301	I	InfiniteSeries	2	3
	II	RiemannIntegration		
	III	ApplicationsofIntegrationsand Improper Integrals		
Linear Algebra I				
USMT 302 ,UAMT 302	I	SystemofEquationsandMatrices	2	3
	II	VectorSpacesoverIR		
	III	Determinants,LinearEquations(Revisited)		
ORDINARY DIFFERENTIAL EQUATIONS				
USMT 303	I	HigherOrderlinearDifferentialEquations	2	3
	II	SystemsofFirstOrder Linear differential equations		
	III	NumericalSolutionsofOrdinary Differential Equations		
PRACTICALS				
USMTP03		Practicals based on USMT301, USMT 302 and USMT 303	3	5
UAMTP03		Practicals based on UAMT301, UAMT 302	2	4

SEMESTER IV

Course Code	UNIT	TOPICS	Credits	L/Week
Multivariable Calculus I				
USMT 401, UAMT 401	I	Functionsofseveralvariables	2	3
	II	DifferentiationofScalarFields		
	III	ApplicationsofDifferentiationof Scalar Fields and Differentiation of Vector Fields		
Linear Algebra II				
USMT 402 ,UAMT 402	I	Lineartransformation,Isomorphism, Matrix associated with L.T.	2	3
	II	Innerproductspaces		
	III	Eigenvalues,eigenvectors, diagonalizable matrix		
Numerical methods (Elective A)				
USMT 403A	I	Solutionsofalgebraicand transcendental equations	2	3
	II	Interpolation,Curvefitting, Numerical integration		
	III	Solutionsoflinearsystem of Equations and eigen value problems		
Statistical methods an their applications(Elective B)				
USMT 403B	I	DescriptiveStatisticsand random variables	2	3
	II	ProbabilityDistributionand Correlation		
	III	InferentialStatistics		
PRACTICALS				
USMTP04		Practicals based on USMT401, USMT 402 and USMT 403	3	5
UAMTP04		Practicals based on UAMT401, UAMT 402	2	4

5. Teaching Pattern for Semester III & IV

Teaching Pattern for Semester III

1. Three lectures per week per course. Each lecture is of 48 minutes duration.

2. One Practical (2L) per week per batch for courses USMT301, USMT 302 combined and one Practical (3L) per week for course USMT303 (the batches tobe formed as prescribed by the University. Each practical session is of 48 minutes duration.)

Teaching Pattern for Semester IV

1. Three lectures per week per course. Each lecture is of 48 minutes duration.

2. One Practical (2L) per week per batch for courses USMT301, USMT 302 combined and one Practical (3L) per week for course USMT303 (the batches to be formed as prescribed by the University. Each practical session is of 48 minutes duration.)

6. Consolidated Syllabus for Semester III & IV

Semester-I I I

Note: Unless indicated otherwise, proofs of the results mentioned in the syllabus should be covered.

USMT301/ UAMT301: Calculus III

Unit I. Infinite Series (15 Lectures)

1. Infinite series in R. Definition of convergence an
∑ddivergence.Basicexamplesincluding∞geometricseries.Elementaryresultssuchasif

converse not true. Cauchy Criterion. Algebra of convergent series.

2. Tests for convergence: Comparison Test, Limit Comparison Test, Ratio Test (without proof), Root Test (without proof), Abel Test (without proof) and Dirichlet
∑Test(without∞1proof).Examples.Thedecimalexpansionofrealnumbers.Convergenceof
npn=1∞1Divergenceofharmonicseries
∑
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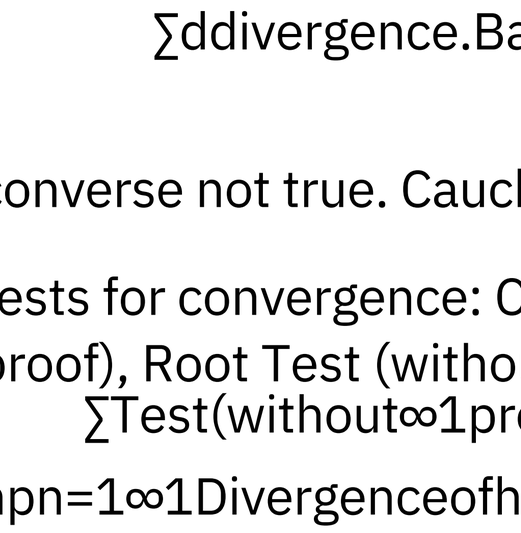
3. Alternating series. Leibnitz's Test. Examples. Absolute convergence, absolute convergence implies convergence but not conversely. Conditional Convergence.



Unit II. Riemann Integration (15 Lectures)

1. Idea of approximating the area under a curve by inscribed and circumscribed rectangles. Partitions of an interval. Refinement of a partition. Upper and Lower sums for a bounded real valued function on a closed and bounded interval. Riemann integrability and the Riemann integral.

2. Criterion for Riemann integrability. Characterization of the Riemann integral as the limit of a sum. Examples.

3. Algebra of Riemann in R_b $-\to b$ $f(x)dx.(ii)\left|\int_a^b f(x)dx\right| \leqslant \int_a^b |f|(x)dx(iii)If f(x)\geq 0$ integrablefunctions.Also,basicresultssuchasiff:[a,b] $\int_a^b f(x)dx = \int_a^c f(x)dx +$ $f(x)dx \geq 0.a$ integrable and $f(x)dx$ $\int_a^b f(x)dx$ $f(x)dx \geq 0 forallx \in [a,b]then$ a4.Riemannintegrabilityofacontinuousfunction,andmoregenerallyofaboundedfunction whose set of discontinuities has only finitely many points. Riemann integrability of monotone functions.

Unit III. Applications of Integrations and Improper Integrals (15 lectures)

1. Area between the two curves. Lengths of plane curves. Surface area of surfaces of revolution.

$\int$x2.ContinuityofthefunctionF(x)= $f(t)dt, x \in [a,b]$,whenf:[a,b]$-\to$RisRiemanna integrable. First and Second Fundamental Theorems of Calculus.

3. Mean value theorem. Integration by parts formula. Leibnitz's Rule.

4. Definition of two types of improper integrals. Necessary and sufficient conditions for convergence.

5. Absolute convergence. Comparison and limit comparison tests for convergence.

6. Gamma and Beta functions and their properties. Relationship between them (without pro of).

Reference Books

1. Sudhir Ghorpade, Balmohan Limaye; A Course in Calculus and Real Analysis (second edition); Springer.

2. R.R. Goldberg; Methods of Real Analysis; Oxford and IBH Pub. Co., New Delhi, 1970.

3. Calculus and Analytic Geometry (Ninth Edition); Thomas and Finney; Addison-Wesley, Reading Mass., 1998.

4. T. Apostol; Calculus Vol. 2; John Wiley.

Additional Reference Books

1. Ajit Kumar, S.Kumaresan; A Basic Course in Real Analysis; CRC Press, 2014

2. D. Somasundaram and B.Choudhary; A First Course in Mathematical Analysis, Narosa, New Delhi, 1996.

3. K. Stewart; Calculus, Brooke/Cole Publishing Co, 1994.

4. J. E. Marsden, A.J. Tromba and A. Weinstein; Basic Multivariable Calculus; Springer.

5. R.G. Brtle and D. R. Sherbert; Introduction to Real Analysis Second Ed. ; John Wiley, New Yorm, 1992.

6. M. H. Protter; Basic Elements of Real Analysis; Springer-Verlag, New York, 1998.

USMT/UAMT 302: Linear Algebra I

Unit I. System of Equations, Matrices (15 Lectures)

1. Systems of homogeneous and non-homogeneous linear equations, Simple examples of find- ing solutions of such systems. Geometric and algebraic understanding of the solutions. Matrices (with real entries), Matrix representation of system of homogeneous and non-homogeneous linear equations. Algebra of solutions of systems of homogeneous linear equations. A system of homogeneous linear equations with number of unknowns more than the number of equations has infinitely many solutions.

2. Elementary row and column operations. Row equivalent matrices. Row reduction (of a matrix to its row echelon form). Gaussian elimination. Applications to solving systems of linear equations. Examples.

3. Elementary matrices. Relation of elementary row operations with elementary matrices. Invertibility of elementary matrices. Consequences such as (i) a square matrix is invertible if and only if its row echelon form is invertible. (ii) invertible matrices are products of elementary matrices. Examples of the computation of the inverse of a matrix using Gauss elimination method.

Unit II. Vector space over R (15 Lectures)

1. Definition of a vector space over R. Subspaces; criterion for a nonempty subset to be a subspace of a vector space. Examples of vector spaces, including the Euclidean space Rn, lines, planes and hyperplanes in Rn passing through the origin, space of systems of homogeneous linear equations, space of polynomials, space of various types of matrices, space of real valued functions on a set.

2. Intersections and sums of subspaces. Direct sums of vector spaces. Quotient space of a vector space by its subspace.

3. Linear combination of vectors. Linear span of a subset of a vector space. Definition of a finitely generated vector space. Linear dependence and independence of subsets of a vector space.

4. Basis of a vector space. Basic results that any two bases of a finitely generated vector space have the same number of elements. Dimension of a vector space. Examples. Bases of a vector space as a maximal linearly independent sets and as minimal generating sets.

Unit III. Determinants, Linear Equations (Revisited) (15 Lectures)

1. Inductive definition of the determinant of a n × n matrix (e. g. in terms of expansion along the first row). Example of a lower triangular matrix. Laplace expansions along an (arbitrary row or column). Determinant expansions using permutations

$$\det(A) = \sum_{\sigma \in S_n} \operatorname{sign}(\sigma) \prod_{i=1}^{n} a_{\sigma(i),i}$$

2. Basic properties of determinants (Statements only); (i) det A = det AT . (ii) Multilinearity and alternating property for columns and rows. (iii) A square matrix A is invertible if and only if detA 6= 0. (iv) Minors and cofactors. Formula for A−1 when detA = 0. (v) det(AB) = det A det B.

3. Row space and the column space of a matrix as examples of vector space. Notion of row rank and the column rank. Equivalence of the row rank and the column rank. Invariance of rank upon elementary row or column operations. Examples of computing the rank using row reduction.

4. Relation between the solutions of a system of non-homogeneous linear equations and the associated system of homogeneous linear equations. Necessary and sufficient condition for a system of non-homogeneous linear equations to have a solution [viz., the rank of the coefficient matrix equals the rank of the augmented matrix [A |B]]. Equivalence of statements (in which A denotes an n × n matrix) such as the following.

(i) The system Ax = b of non-homogeneous linear equations has a unique solution.

(ii) The system Ax = 0 of homogeneous linear equations has no nontrivial solution.

(iii) A is invertible.

(iv) detA 6=0.

(v) rank(A) = n.

5. Cramers Rule. LU Decomposition. If a square matrix A is a matrix that can be reduced to row echelon form U by Gauss elimination without row interchanges, then A can be factored as A = LU where L is a lower triangular matrix.

Reference books

1 Howard Anton, Chris Rorres, Elementary Linear Algebra, Wiley Student Edition).

2 Serge Lang, Introduction to Linear Algebra, Springer.

3 S Kumaresan, Linear Algebra - A Geometric Approach, PHI Learning.

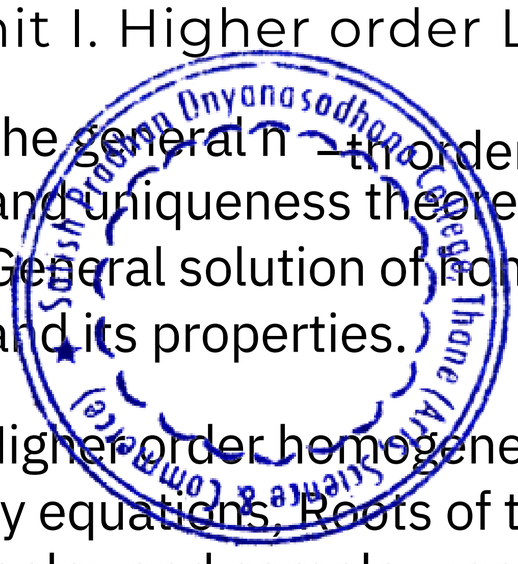
4 Sheldon Axler, Linear Algebra done right, Springer.

5 Gareth Williams, Linear Algebra with Applications, Jones and Bartlett Publishers.

6 David W. Lewis, Matrix theory.

USMT303: Ordinary Differential Equations

Unit I. Higher order Linear Differential equations (15 Lectures)



1. The general n −th order linear differential equations, Linear independence, An existence and uniqueness theorem, the Wronskian, Classification: homogeneous and non-homogeneous, General solution of homogeneous and non-homogeneous LDE, The Differential operator and its properties.

2. Higher order homogeneous linear differential equations with constant coefficients, the auxiliary equations, Roots of the auxiliary equations: real and distinct, real and repeated, complex and complex repeated.

3. Higher order homogeneous linear differential equations with constant coefficients, the method of undermined coefficients, method of variation of parameters.

4. The inverse differential operator and particular integral, Evaluation of $\frac{1}{f(D)}$ for the functions like eax, sin ax, cos ax, xm, xm sin ax, xm cos ax, eaxV and xV where V is any function ofx,

5. Higher order linear differential equations with variable coefficients:

The Cauchy's equation: $x^3\frac{d^3y}{dx^3} + x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = f(x)$ and

The Legendre's equation: $(ax+b)^3\frac{d^3y}{dx^3} + (ax+b)^2\frac{d^2y}{dx^2} + (ax+b)\frac{dy}{dx} + y = f(x)$.

Reference Books

1. Units 5, 6, 7 and 8 of E.D. Rainville and P.E. Bedient; Elementary Differential Equations; Macmillan.

2. Units 5, 6 and 7 of M.D. Raisinghania; Ordinary and Partial Differential Equations; S. Chand.

Unit II. Systems of First Order Linear Differential Equations (15 Lectures)

(a) Existence and uniqueness theorem for the solutions of initial value problems for a system of two first order linear differential equations in two unknown functions x,y of a single independent variable t, of the form $\frac{dx}{dt} = F(t, x, y)$, $\frac{dy}{dt} = G(t, x, y)$ (Statementonly).

(b) Homogeneous linear system of two first order differential equations of a single independent variable, of the form $\frac{dx}{dt} = a_1(t)x + b_1(t)y$, $\frac{dy}{dt} = a_2(t)x + b_2(t)y$. (c) Wronskian for a homogeneous linear system of first order linear differential equations in two functions x, y of a single independent variable t. Vanishing properties of the Wronskian. Relation with linear independence of solutions.

(d) Homogeneous linear systems with constant coefficients in two unknown functions x,y of a single independent variable t. Auxiliary equation associated to a homogenous system of equations with constant coefficients. Description fo the general solution depending on the roots and their multiplicities of the auxiliary equation, proof of independence of the solutions. Real form of solutions in case the auxiliary equation has complex roots.

(e) Non-homogeneous linear system of linear system of two first order differential equations in two unknown functions of a single independent variable t, of the form $\frac{dx}{dt} = a_1(t)x + b_1(t)y + f_1(t)$, $\frac{dy}{dt} = a_2(t)x + b_2(t)y + f_2(t)$.

General Solution of non-homogeneous system. Relation between the solutions of a system

of non-homogeneous linear differential equations and the associated system of homogeneous linear differential equations.

Reference Books

1. G.F. Simmons; Differential Equations with Applications and Historical Notes; Taylor's and Francis.

Unit III. Numerical Solution of Ordinary Differential Equations (15 lectures)

1. Numerical Solution of initial value problem of first order ordinary differential equation using:

   (i) Taylor's series method,

   (ii) Picard's method for successive approximation and its convergence,

   (iii) Euler's method and error estimates for Euler's method,

   (iv) Modified Euler's Method,

   (v) Runge-Kutta method of second order and its error estimates,

   (vi) Runge-Kutta fourth order method.

2. Numerical solution of simultaneous and higher order ordinary differential equation using:

   (i) Runge-Kutta fourth order method for solving simultaneous ordinary differential equation,

   (ii) Finite difference method for the solution of two point linear boundary value problem.

Reference Books

1. Units 8 of S. S. Sastry, Introductory Methods of Numerical Analysis, PHI.

Additional Reference Books

1. E.D. Rainville and P.E. Bedient, Elementary Differential Equations, Macmillan.

2. M.D. Raisinghania, Ordinary and Partial Differential Equations, S. Chand.

3. G.F. Simmons, Differential Equations with Applications and Historical Notes, Taylor's and Francis.

4. S. S. Sastry, Introductory Methods of Numerical Analysis, PHI.

5. K. Atkinson, W.Han and D Stewart, Numerical Solution of Ordinary Differential Equations, Wiley.

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USMT P03 / UAMT P03: Practicals

Suggested Practicals for USMT 301/ UAMT 301

1. Examples of convergent / divergent series and algebra of convergent series.
2. Tests for convergence of series.
3. Calculation of upper sum, lower sum and Riemann integral.
4. Problems on properties of Riemann integral.
5. Problems on fundamental theorem of calculus, mean value theorems, integration by parts, Leibnitz rule.
6. Convergence of improper integrals, different tests for convergence. Beta Gamma Functions.
7. Miscellaneous Theoretical Questions based on full paper.

Suggested Practicals for USMT302 / UAMT 302

1. Systems of homogeneous and non-homogeneous linear equations.
2. Elementary row/column operations and Elementary matrices.
3. Vector spaces, Subspaces.
4. Linear Dependence/independence, Basis, Dimension.
5. Determinant and Rank of a matrix.
6. Solution to a system of linear equations, LU decomposition
7. Miscellaneous Theory Questions.
8. Miscellaneous theory questions from units I, II and III.

Suggested Practicals For USMT 303

1. Finding the general solution of homogeneous and non-homogeneous higher order linear differential equations.
2. Solving higher order linear differential equations using method of undetermined coefficients and method of variation of parameters.
3. Solving a system of first order linear ODES have auxiliary equations with real and complex ro ots.
4. Finding the numerical solution of initial value problems using Taylor's series method, Picard's method, modified Euler's method, Runge-Kutta method of fourth order and calculating their accuracy.
5. Finding the numerical solution of simultaneous ordinary differential equation using fourth order Runge-Kutta method.
6. Finding the numerical solution of two point linear boundary value problem using Finite difference method.

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Semester-IV

Note: Unless indicated otherwise, proofs of the results mentioned in the syllabus should be covered.

USMT 401/ UAMT 401: Multivariable Calculus I

UNIT I. Functions of Several Variables (15 Lectures)

1. Review of vectors in Rn [with emphasis on R2 and R3] and basic notions such as addition and scalar multiplication, inner product, length (norm), and distance between two points.

2. Real-valued functions of several variables (Scalar fields). Graph of a function. Level sets (level curves, level surfaces, etc). Examples. Vector valued functions of several variables (Vector fields). Component functions. Examples.

3. Sequences, Limits and Continuity: Sequence in Rn [with emphasis on R2 and R3] and their limits. Neighbourhoods in Rn. Limits and continuity of scalar fields. Composition of continuous functions. Sequential characterizations. Algebra of limits and continuity (Results with proofs). Iterated limits.
Limits and continuity of vector fields. Algebra of limits and continuity vector fields. (without proofs).

4. Partial and Directional Derivatives of scalar fields: Definitions of partial derivative and directional derivative of scalar fields (with emphasis on R2 and R3). Mean Value Theorem of scalar fields.

UNIT II. Differentiation of Scalar Fields (15 Lectures)

1. Differentiability of scalar fields (in terms of linear transformation). The concept of (total) derivative. Uniqueness of total derivative of a differentiable function at a point. Examples of functions of two or three variables. Increment Theorem. Basic properties including (i) continuity at a point of differentiability, (ii)existence of partial derivatives at a point of differentiability, and (iii) differentiability when the partial derivatives exist and are continuous.

2. Gradient. Relation between total derivative and gradient of a function. Chain rule.
Geometric properties of gradient. Tangent planes.

3. Euler's Theorem.

4. Higher order partial derivatives. Mixed Partial Theorem (n=2).

UNIT III. Applications of Differentiation of Scalar Fields and Differentiation of Vector Fields (15 lectures)

1. Applications of Differentiation of Scalar Fields: The maximum and minimum rate of change of scalar fields. Taylor's Theorem for twice continuously differentiable functions. Notions of local maxima, local minima and saddle points. First Derivative Test. Examples. Hessian matrix. Second Derivative Test for functions of two variables. Examples. Method of Lagrange Multipliers.

2. Differentiation of Vector Fields: Differentiability and the notion of (total) derivative. Differentiability of a vector field implies continuity, Jacobian matrix. Relationship between total derivative and Jacobian matrix. The chain rule for derivative of vector fields (state- ments only).

Reference books

1. T. Apostol; Calculus, Vol. 2 (Second Edition); John Wiley.
2. Sudhir Ghorpade, Balmohan Limaye; A Course in Multivariable Calculus and Analysis (Second Edition); Springer.
3. Walter Rudin; Principles of Mathematical Analysis; McGraw-Hill, Inc.
4. J. E. Marsden, A.J. Tromba and A. Weinstein, Basic Multivariable Calculus; Springer.
5. D.Somasundaram and B.Choudhary; A First Course in Mathematical Analysis, Narosa, New Delhi, 1996.
6. K. Stewart; Calculus; Booke/Cole Publishing Co, 1994.

Additional Reference Books

1. Calculus and Analytic Geometry, G.B. Thomas and R. L. Finney, (Ninth Edition); Addison-Wesley,1998.
2. Howard Anton; Calculus- A new Horizon,(Sixth Edition); John Wiley and Sons Inc, 1999.
3. S L Gupta and Nisha Rani; Principles of Real Analysis; Vikas Publishing house PVT LTD.
4. Shabanov, Sergei; Concepts in Calculus, III: Multivariable Calculus; University Press of Florida, 2012.
5. S C Malik and Savita Arora; Mathematical Analysis; New Age International Publishers.

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USMT402/UAMT402: Linear Algebra II

UNIT I. Linear Transformations

1. Definition of a linear transformation of vector spaces; elementary properties. Examples. Sums and scalar multiples of linear transformations. Composites of linear transformations. A Linear transformation of $V \longrightarrow W$, where V,W are vector spaces over R and V is a finite-dimensional vector space is completely determined by its action on an ordered basis ofV.

2. Null-space (kernel) and the image (range) of a linear transformation. Nullity and rank of a linear transformation. Rank-Nullity Theorem (Fundamental Theorem of Homomorphisms).

3. Matrix associated with linear transformation of $V \longrightarrow W$ where V and W are finite dimensional vector spaces over R.. Matrix of the composite of two linear transformations. Invertible linear transformations (isomorphisms), Linear operator, Effect of change of bases on matrices of linear operator.

4. Equivalence of the rank of a matrix and the rank of the associated linear transformation. Similar matrices.

UNIT II. Inner Products and Orthogonality

1. Inner product spaces (over R). Examples, including the Euclidean space Rn and the space of real valued continuous functions on a closed and bounded interval. Norm associated to an inner product. Cauchy-Schwarz inequality. Triangle inequality.

2. Angle between two vectors. Orthogonality of vectors. Pythagoras theorem and some geometric applications in R2. Orthogonal sets, Orthonormal sets. Gram-Schmidt orthogonalizaton process. Orthogonal basis and orthonormal basis for a finite-dimensional inner product space.

3. Orthogonal complement of any set of vectors in an inner product space. Orthogonal complement of a set is a vector subspace of the inner product space. Orthogonal decomposition of an inner product space with respect to its subspace. Orthogonal projection of a vector onto a line (one dimensional subspace). Orthogonal projection of an inner product space onto its subspace.

UNIT III. Eigenvalues, Eigenvectors and Diagonalisation

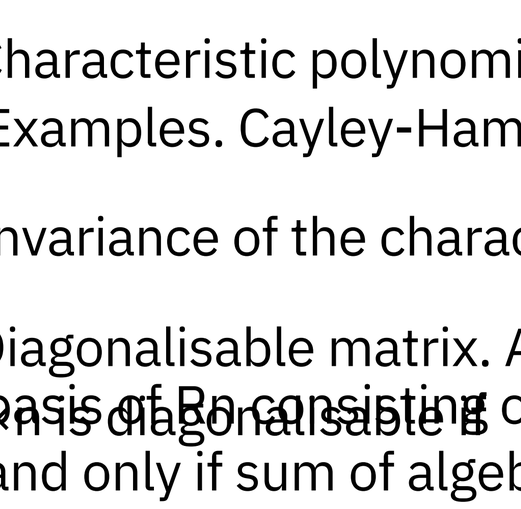
1. Eigenvalues and eigenvectors of a linear transformation of a vector space into itself and of square matrices. The eigenvectors corresponding to distinct eigenvalues of a linear transformation are linearly independent. Eigen spaces. Algebraic and geometric multiplicity of an eigenvalue.

2. Characteristic polynomial. Properties of characteristic polynomials (only statements). Examples. Cayley-Hamilton Theorem. Applications.

3. Invariance of the characteristic polynomial and eigenvalues of similar matrices.

4. Diagonalisable matrix. A real square matrix A is diagonalisable if and only if there is a basis of Rn consisting of eigenvectors of A. (Statement only - An ×n is diagonalisable if and only if sum of algebraic multiplicities is equal to sum of geometric multiplicities of all the eigenvalues of A = n). Procedure for diagonalising a matrix.

5. Spectral Theorem for Real Symmetric Matrices (Statement only). Examples of orthogonal diagonalisation of real symmetric matrices. Applications to quadratic forms and classification of conic sections.



Reference book

1. Howard Anton, Chris Rorres; Elementary Linear Algebra; Wiley Student Edition).

2. Serge Lang; Introduction to Linear Algebra; Springer.

3. S Kumaresan; Linear Algebra - A Geometric Approach; PHI Learning.

4. Sheldon Axler; Linear Algebra done right; Springer.

5. Gareth Williams; Linear Algebra with Applications; Jones and Bartlett Publishers.

6. David W. Lewis; Matrix theory.

USMT403A: Numerical Methods (Elective A)

Unit I. Solution of Algebraic and Transcendental Equations (15L)

1. Measures of Errors: Relative, absolute and percentage errors, Accuracy and precision: Accuracy to n decimal places, accuracy to n significant digits or significant figures, Rounding and Chopping of a number, Types of Errors: Inherent error, Round-off error and Trunca- tion error.

2. Iteration methods based on first degree equation: Newton-Raphson method. Secant method. Regula-Falsi method.
Derivations and geometrical interpretation and rate of convergence of all above methods to be covered.

3. General Iteration method: Fixed point iteration method.

Unit II. Interpolation, Curve fitting, Numerical Integration(15L)

1. Interpolation: Lagrange's Interpolation. Finite difference operators: Forward Difference operator, Backward Difference operator. Shift operator. Newton's forward difference interpolation formula. Newton's backward difference interpolation formula.
Derivations of all above methods to be covered.

2. Curve fitting: linear curve fitting. Quadratic curve fitting.

3. Numerical Integration: Trapezoidal Rule. Simpson's 1/3 rd Rule. Simpson's 3/8th Rule.
Derivations all the above three rules to be covered.

Unit III. Solution Linear Systems of Equations, Eigenvalue problems(15L)

1. Linear Systems of Equations: LU Decomposition Method (Dolittle's Method and Crout's Method). Gauss-Seidel Iterative method.

2. Eigenvalue problems: Jacobi's method for symmetric matrices. Rutishauser method for arbitrary matrices.

Reference Books:

1. Kendall E. and Atkinson; An Introduction to Numerical Analysis; Wiley.

2. M. K. Jain, S. R. K. Iyengar and R. K. Jain; Numerical Methods for Scientific and Engineering Computation; New Age International Publications.

3. S. Sastry; Introductory methods of Numerical Analysis; PHI Learning.

4. An introduction to Scilab-Cse iitb.

Additional Reference Books

1. S.D. Comte and Carl de Boor; Elementary Numerical Analysis, An algorithmic approach; McGrawHillll International Book Company.

2. Hildebrand F.B.; Introduction to Numerical Analysis; Dover Publication, NY.

3. Scarborough James B.; Numerical Mathematical Analysis; Oxford University Press, New Delhi.

USMT403B Statistical Methods and their Applications (Elective B)

Unit I. Descriptive Statistics and random variables (15 Lectures)

Measures of location (mean, median, mode), Partition values and their graphical locations, measures of dispersion, skewness and kurtosis, Exploratory Data Analysis (Five number summary, Box Plot, Outliers), Random Variables (discrete and continuous), Expectation and variance of a random variable.

Unit II. Probability Distributions and Correlation (15 Lectures)

Discrete Probability Distribution (Binomial, Poisson), Continuous Probability Distribution: (Uniform, Normal), Correlation, Karl Pearson's Coefficient of Correlation, Concept of linear Regression, Fitting of a straight line and curve to the given data by the method of least squares, relation between correlation coefficient and regression coefficients.

Unit III. Inferential Statistics (15 lectures)

Population and sample, parameter and statistic, sampling distribution of Sample mean and Sample Variance, concept of statistical hypothesis, critical region, level of significance, confidence interval and two types of errors, Tests of significance (t-test, Z-test, F-test, Chi-Square Test (only applications))

Reference Books

1. FundamentalsofMathematicalStatistics,12thEdition,S.C.GuptaandV.K.Kapoor,Sultan Chand & Sons, 2020.

2. Statistics for Business and Economics, 11th Edition, David R. Anderson, Dennis J. Sweeney and Thomas A. Williams, Cengage Learning, 2011.

3. Introductory Statistics, 8th Edition, Prem S. Mann, John Wiley & Sons Inc., 2013.

4. A First Course in Statistics, 12th Edition, James McClave and Terry Sincich, Pearson Education Limited, 2018.

5. Introductory Statistics, Barbara Illowsky, Susan Dean and Laurel Chiappetta, OpenStax, 2013.

6. Hands-On Programming with R, Garrett Grolemund, O'Reilly.

USMT P04 / UAMT P04: Practicals

Suggested Practical for USMT 401/ UAMT 401

1. Limits and continuity of scalar fields and vector fields, using ”definition and otherwise“ , iterated limits.
2. Computing directional derivatives, partial derivatives and mean value theorem of scalar fields.
3. Differentiability of scalar field,Total derivative, gradient, level sets and tangent planes.
4. Chain rule, higher order derivatives and mixed partial derivatives of scalar fields.
5. Maximum and minimum rate of change of scalar fields. Taylor’s Theorem. Finding Hessian/Jacobean matrix. Differentiation of a vector field at a point. Chain Rule for vector fields.
6. Finding maxima, minima and saddle points. Second derivative test for extrema of functions of two variables and method of Lagrange multipliers.
7. Miscellaneous Theoretical Questions based on full paper.

Suggested Practicals for USMT402/UAMT 402

1. Linear transformation, Kernel, Rank-Nullity Theorem.
2. Linear Isomorphism, Matrix associated with Linear transformations.
3. Inner product and properties, Projection, Orthogonal complements.
4. Orthogonal, orthonormal sets, Gram-Schmidt orthogonalisation
5. Eigenvalues, Eigenvectors, Characteristic polynomial. Applications of Cayley Hamilton Theorem.
6. Diagonalisation of matrix, orthogonal diagonalisation of symmetric matrix and application to quadratic form.
7. Miscellaneous Theoretical Questions based on full paper.

Suggested Practicals for USMT403A

The Practical no. 1 to 6 should be performed either using non-programable scientific calculators or by using the software Scilab.

1. Newton-Raphson method, Secant method.
2. Regula-Falsi method, Iteration Method..
3. Interpolating polynomial by Lagrange’s Interpolation, Newton forward and backward difference Interpolation.
4. Curve fitting, Trapezoidal Rule, Simpson’s 1/3rd Rule, Simpson’s 3/8th Rule.
5. LU decomposition method, Gauss-Seidel Interative method.

6. Jacobi's method, Rutishauser method..

7. Miscellaneous theoretical questions from all units.

Suggested Practicals for USMT403B

All practicals should be performed using any one of the following softwares: MS Excel, R, Strata, SPSS, Sage Math to carry out data analysis and computations.

1. Descriptive Statistics.

2. Random Variables.

3. Probability Distributions.

4. Correlation and Regression.

5. Testing of hypothesis.

6. Case studies.

7. Miscellaneous Theory questions based on Unit I,II,III.

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7. Scheme of Examination (75:25)

The performance of the learners shall be evaluated into two parts.

Internal Assessment of 25 percent marks.
Semester End Examinations of 75 percent marks.

I. Internal Evaluation of 25 Marks:

S.Y.B.Sc. :

(i) One class Test of 20 marks to be conducted during Practical session.

Paper pattern of the Test:

Q1: Definitions/ Fill in the blanks/ True or False with Justification (04 Marks).

Q2: Multiple choice 5 questions. (10 Marks: 5 ×2)

Q3: Attempt any 2 from 3 descriptive questions. (06 marks: 2 ×3)

(ii) Active participation in routine class: 05 Marks.

OR

Students who are willing to explore topics related to syllabus, dealing with applications historical development or some interesting theorems and their applications can be encouraged to submit a project for 25 marks under the guidance of teachers.

S.Y.B.A. :

(i) One class Test of 20 marks to be conducted during Tutorial session.

Paper pattern of the Test:

Q1: Definitions/ Fill in the blanks/ True or False with Justification (04 Marks).

Q2: Multiple choice 5 questions. (10 Marks: 5 ×2)

Q3: Attempt any 2 from 3 descriptive questions. (06 marks: 2 × 3)

(ii) Journal : 05 Marks.

OR

Students who are willing to explore topics related to syllabus, dealing with applications historical development or some interesting theorems and their applications can be encouraged to submit a project for 25 marks under the guidance of teachers.

II. Semester End Theory Examinations : There will be a Semester-end external Theory examination of 75 marks for each of the courses USMT301/UAMT301, USMT/USAT 302, USMT 303 of Semester III and USMT/UAMT401, USMT/UAMT 402, USMT 403 of semester IV to be conducted by the college.

1. Duration: The examinations shall be of 2 and 1 2 hours duration.

2. Theory Question Paper Pattern:

a) There shall be FOUR questions. The first three questions Q1, Q2, Q3 shall be of 20 marks, each based on the units I, II, III respectively. The question Q4 shall be of 15 marks based on the entire syllabus.

b) All the questions shall be compulsory. The questions Q1, Q2, Q3, Q4 shall have internal choices within the questions. Including the choices, the marks for each question shall be 25-27.

c) The questions Q1, Q2, Q3, Q4 may be subdivided into sub-questions as a, b, c, d & e, etc and the allocation of marks depends on the weightage of the topic.

III. Semester End Examinations Practicals:

At the end of the Semesters III & IV Practical examinations of three hours duration and 150 marks shall be conducted for the courses USMTP03, USMTP04.

At the end of the Semesters III & IV Practical examinations of two hours duration and 100 marks shall be conducted for the courses UAMTP03, UAMTP04.
In semester III, the Practical examinations for USMT301/UAMT301, USMT302/UAMT302 and USMT303 are held together by the college.
In Semester IV, the Practical examinations for USMT401/UAMT401, USMT402/UAMT402 and USMT403 are held together by the college.

Paper pattern: The question paper shall have two parts A and B.
Each part shall have two Sections.

Section I Objective in nature: Attempt any Eight out of Twelve multiple choice questions (04 objective questions from each unit) (8 × 3 = 24 Marks).

Section II Problems: Attempt any Two out of Three (01 descriptive question from each unit) (8 × 2 = 16 Marks).

Practical Course	Part A	Part B	PartC	Marks out of	duration
USMTP03	Questions from USMT301	Questions from USMT302	Questions from USMT 303	120	3 hours
UAMTP03	Questions from UAMT301	Questions from UAMT302	——	80	2 hours
USMTP04	Questions from USMT401	Questions from USMT402	Questions from USMT403	120	3 hours
UAMTP04	Questions from UAMT401	Questions from UAMT402	——	80	2 hours

Marks for Journals and Viva:

For each course USMT301/UAMT301, USMT302/UAMT302, USMT303, USMT401/UAMT401, USMT402/UAMT402, USMT3031:

1. Journal: 10 marks (5 marks for each journal).

2. Viva: 10 marks.

Each Practical of every course of Semester III and IV shall contain 10 (ten) problems out of which minimum 05 (five) have to be written in the journal. .
A student must have a certified journal before appearing for the practical examination.
In case a student does not posses a certified journal he/she will be evaluated for 120/80 marks.
He/she is not qualified for Journal + Viva marks.

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